

November 17

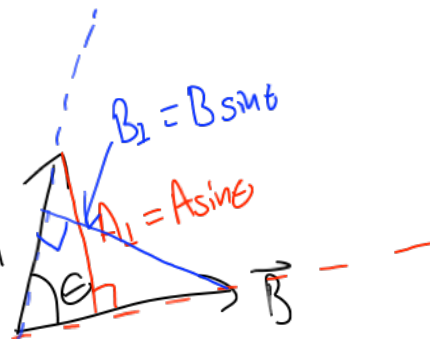
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Cross products

$$\vec{C} = \vec{A} \times \vec{B}$$

Fingers stick out
along this

curl to



$$= \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$

$$= |\vec{A}| |\vec{B}| \sin \theta$$

$$= A_{\perp} |\vec{B}|$$

$$= |\vec{A}| B_{\perp}$$

} → RHR for direction

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

Q1:

What is the direction of $\langle 0, 0, 3 \rangle \times \langle 0, 4, 0 \rangle$?	A) +x B) -x C) +y D) -z E) zero magnitude
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$$\hat{z} \times \hat{y} = -\hat{x}$$

$$A_y B_z - A_z B_y = 0 \cdot 0 - 3 \cdot 4 = -12$$

$$A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

$$\begin{aligned} \hat{x} \times \hat{y} &= \hat{z} = -\hat{y} \times \hat{x} \\ \hat{y} \times \hat{z} &= \hat{x} = -\hat{z} \times \hat{y} \\ \hat{z} \times \hat{x} &= \hat{y} = -\hat{x} \times \hat{z} \end{aligned}$$

Q2:

What is the direction of

$\langle 0, 4, 0 \rangle \times \langle 0, 0, 3 \rangle$?

$$\hat{y} \times \hat{z} = \hat{x}$$

- A) +x
- B) -x
- C) +y
- D) -z
- E) zero magnitude

Q3:

What is the direction of
 $\langle 0, 0, 6 \rangle \times \langle 0, 0, -3 \rangle$?

$$\hat{z} \times (-\hat{z})$$

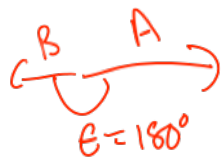
A) +x

B) -x

C) +y

D) -z

E) zero magnitude



$$\sin(180^\circ) = 0$$

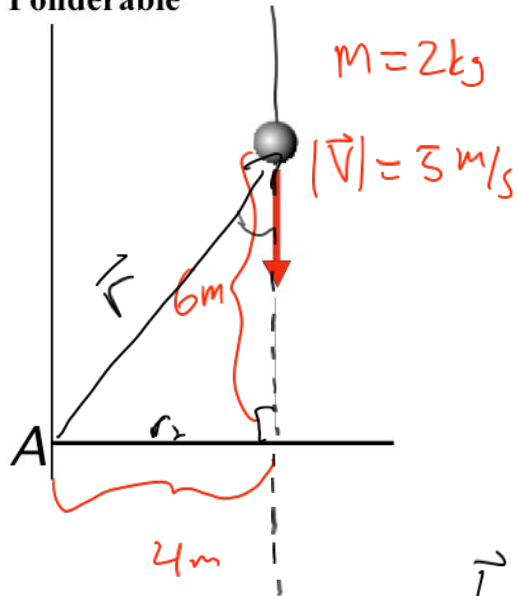
$$\hat{x} \times \hat{x} = 0$$

$$\hat{y} \times \hat{y} = 0$$

$$\hat{z} \times \hat{z} = 0$$

crossproduct.py

Ponderable



What is angular
momentum about
point A?

$$\vec{L} = \vec{r} \times \vec{p}$$

$$|\vec{L}| = |\vec{r}| |\vec{p}| \sin \theta = r_{\perp} p = 40 \text{ kg m}^2/\text{s}$$

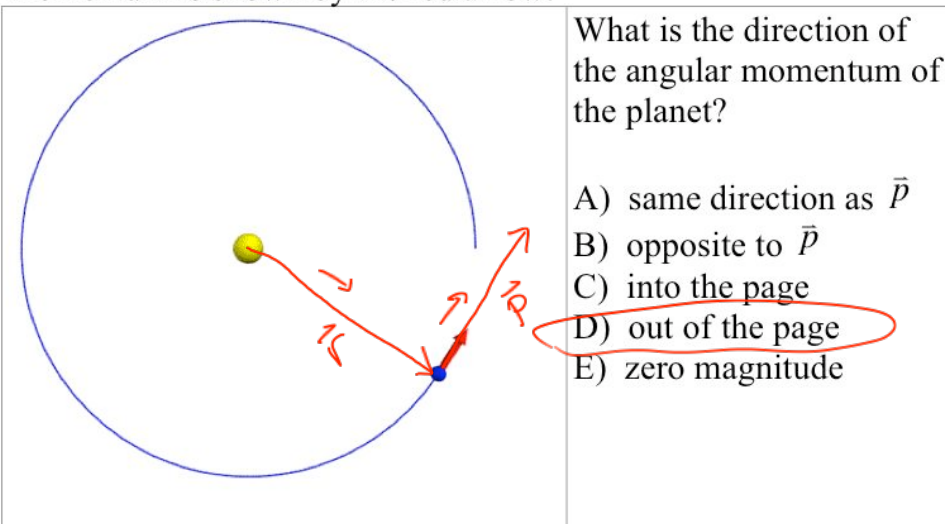
$$r_{\perp} = 4 \text{ m} \quad p = 10 \text{ kg m/s}$$

$$\vec{L} = \langle 4 \text{ m}, 6 \text{ m}, 0 \rangle \times \langle 0, -10 \text{ kg m/s}, 0 \rangle$$

$$= \langle 0, 0, -40 \text{ kg m}^2/\text{s} \rangle$$

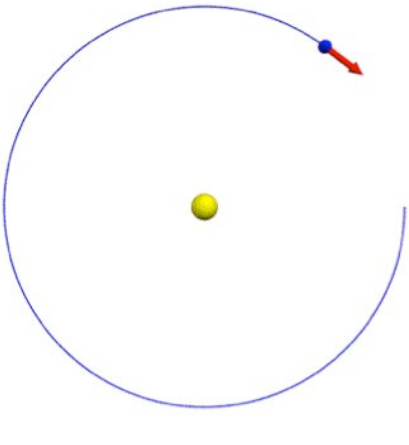
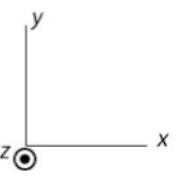
by RHR $-\hat{z}$

Q4: A planet orbits a star, in a circular orbit in the xy plane. Its momentum is shown by the red arrow.

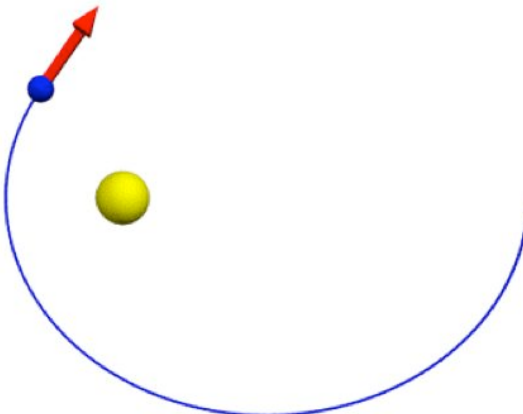


grab onto axis with right hand,
with fingers curling in direction of motion,
Thumb points in dir of $\vec{L}$.

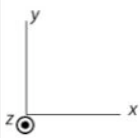
Q5: A planet orbits a star, in a circular orbit in the xy plane. Its momentum is shown by the red arrow. What is its angular momentum?

	A) $+x$ B) $-y$ C) $+z$ D) $-z$ E) zero magnitude	 (z-axis points out of page)
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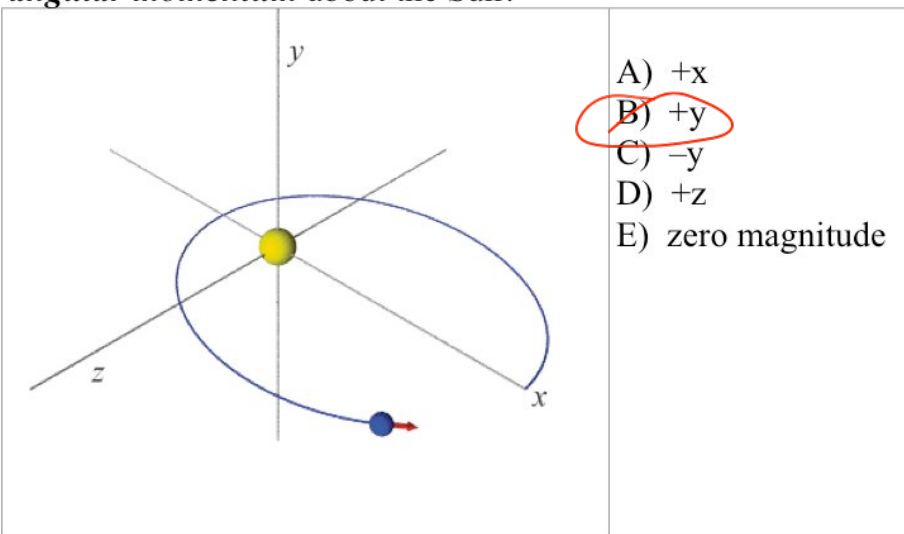
Q6: A comet orbits the Sun, in the xy plane. Its momentum is shown by the red arrow.
What is the direction of the comet's *angular momentum* about the Sun?

	A) same direction as $\vec{p}$ B) opposite to $\vec{p}$ C) into the page D) out of the page E) zero magnitude
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Q7: A comet orbits the Sun, in the xy plane. Its momentum is shown by the red arrow. What is the direction of the comet's *angular momentum* about the Sun?

	A) $-x$ B) $+y$ C) $-y$ D) $+z$ E) zero magnitude	 (z-axis points out of page)
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Q8: A comet orbits the Sun in the xz plane. Its momentum is shown by the red arrow. What is the direction of the comet's *angular momentum* about the Sun?



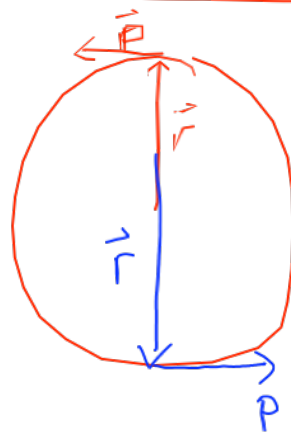
Q9: If an object is traveling at a constant speed in a vertical circle, how does the object's angular momentum change as the object goes from the top of the circle to the bottom of the circle?

A) $|\vec{L}|$ increases.

B) $|\vec{L}|$ decreases.

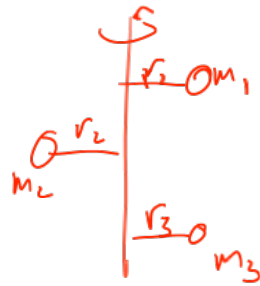
C) $|\vec{L}|$ stays the same, but the direction of $|\vec{L}|$ changes.

D) The direction and magnitude of $|\vec{L}|$ remain the same.



$$I = M_1 r_1^2 + M_2 r_2^2 + \dots$$

What is I_{N_2} ?

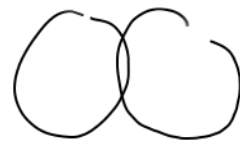


Demo

Ponderable: Moment of Inertia for Nitrogen

$$I = m_1 r_1^2 + m_2 r_2^2 + \dots = \sum_i m_i r_i^2$$

$$I_{N_2} ?$$

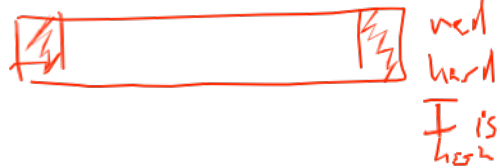


Discussion

$$\begin{aligned}
 I &= M_N r^2 + M_N r^2 = 2 M_N r^2 \\
 &= 2 (2.3 \times 10^{-26} \text{ kg}) (10^{-10} \text{ m})^2 \\
 &= 4.6 \times 10^{-46} \text{ kg m}^2
 \end{aligned}$$

Care about I because it tells us how easy to rotate.

Moment of inertia tells us about distribution of mass



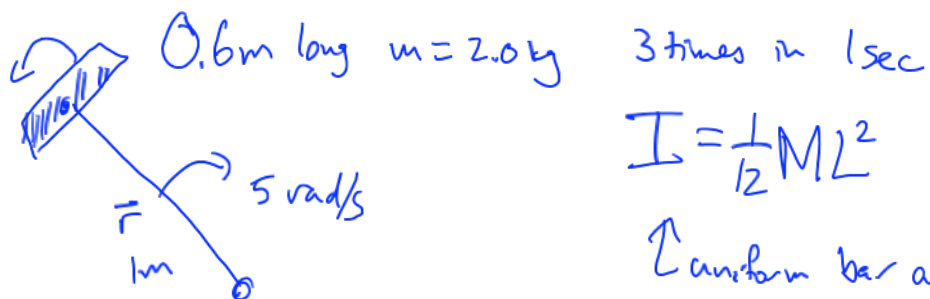
$$|\vec{p}| = m|\vec{v}|$$

$$|\vec{L}| = I|\vec{\omega}|$$

ω is Angular Speed in rad/s

$$K_{\text{tran}} = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$$



$$I = \frac{1}{12} ML^2$$

↑ uniform bar about center

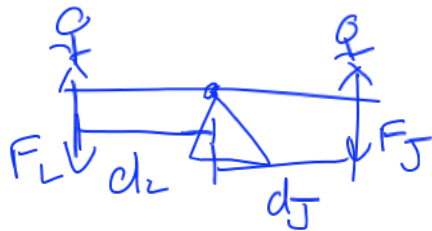
$$\vec{L}_{\text{rot}} = ? = I \omega \odot = \frac{1}{12} ML^2 \omega \odot = \frac{1}{12} (2 \text{ kg}) (0.6 \text{ m})^2 \frac{3 \times 2\pi}{1 \text{ s}} \odot = 1.1 \frac{\text{kg m}^2}{\text{s}} \odot$$

$$\vec{L}_{\text{trans}} = ? = I \omega \otimes = mr^2 \omega \otimes = 2 \text{ kg} (1 \text{ m})^2 \frac{5 \text{ rad}}{\text{s}} \otimes = 10 \frac{\text{kg m}^2}{\text{s}} \otimes$$

$$\vec{L}_{\text{tot}} = ? = \vec{L}_{\text{rot}} + \vec{L}_{\text{trans}} = 8.9 \frac{\text{kg m}^2}{\text{s}} \otimes$$

Torque is combination of position + Force

$$\vec{\tau} = \vec{r} \times \vec{F}$$



$$F_J \neq F_L$$

Want torques to be the same

$$\vec{\tau}_L = F_L d_L \odot \quad \vec{\tau}_J = F_J d_J \otimes \quad d_J = 4m$$

Dr L 200 lbs

J 100 lbs

$$d_L = 2m$$

$d_J = ?$ so balanced?

$$F_L d_L = F_J d_J$$

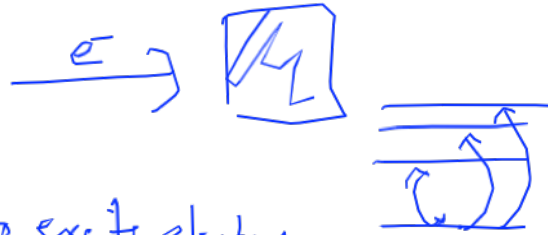
Review

Chapter 7 Energy Quantization

3 processes

1) electron excitation

e^- loses K to excite electrons
in g

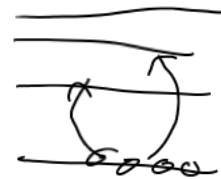


2) Photon emission



3) Photon absorption

↳ dark lines



Chapter 8: Multiparticle Systems

Use point particle system to figure

out K_{trans} , $\vec{P}_{\text{cm}}$ $\nearrow$ all mass at center of
mass, no internal
structure

Use real system to figure out

K_{rot} , K_{vib} , ...

(no vibration,
rotation, etc)

Chapter 9: Collisions

$$\vec{P}_{f, \text{tot}} = \vec{P}_{i, \text{tot}} + \vec{F}_{\text{net, ext}} \Delta t$$

pick our system to be things colliding

$$\text{For collision } \Delta t \text{ small} \Rightarrow \vec{P}_{f, \text{tot}} = \vec{P}_{i, \text{tot}}$$

1) elastic $\rightarrow$ no change in K

2) inelastic $\rightarrow$ change in K

Chapter 10:

$$\vec{L} = \vec{r} \times \vec{p}$$

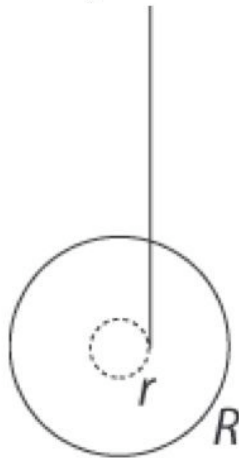
$$|\vec{L}| = I \omega$$

$$I = \sum_i m_i r_i^2$$

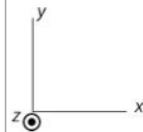
$$\boxed{R + IR}$$

Q1: A yo-yo is in the xy plane. You pull up on the string with a force of magnitude 0.6 N. What is the direction of the torque you exert on the yo-yo?

$r = 0.005 \text{ m}$, $R = 0.035 \text{ m}$



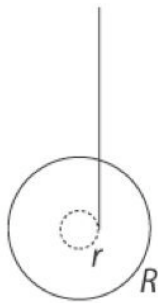
- A) $+x$
- B) $-x$
- C) $+y$
- D) $-y$
- E) $+z$



(z-axis points out of page)

Q2: A yo-yo is in the xy plane. You pull up on the string with a force of magnitude 0.6 N. What is the magnitude of the torque you exert on the yo-yo?

$r = 0.005 \text{ m}$, $R = 0.035 \text{ m}$



- A) $0.003 \text{ N} \cdot \text{m}$
- B) $0.021 \text{ N} \cdot \text{m}$
- C) $0.035 \text{ N} \cdot \text{m}$
- D) $0.6 \text{ N} \cdot \text{m}$
- E) cannot be determined without knowing the length of the string

$$\vec{L}_f = \vec{L}_i + \vec{L}_{\text{net, ext}} \Delta t \quad \text{Angular momentum principle}$$

$\vec{L}_{\text{net, ext}}$ small

$$\Rightarrow \vec{L}_f = \vec{L}_i$$

$\uparrow$ $\vec{L}_i$	$\downarrow$ $\vec{L}_{\text{wheel}}$	$\uparrow$ $\vec{L}_{\text{rotor}}$
$L = I\omega$	$I = mr^2$ $r \downarrow \quad I \downarrow^2$	$K = \frac{1}{2} I \omega^2$ $\omega \uparrow$